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LETTER TO THE EDITOR

Formation and maintenance of complex systems

M A F Gomes†, F A O de Souza† and Sadhan K Adhikari‡

† Departamento de Física, Universidade Federal de Pernambuco 50.670-901 Recife, Pernambuco, Brazil

‡ Instituto de Física Teórica, Universidade Estadual Paulista 01.405-900 São Paulo, São Paulo, Brazil

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Abstract. A complex system is often identified by the absence of a characteristic length, e.g. as in a fractal. A very large system subject to a fragmentation and/or aggregation dynamics passes through such complex configurations. We study statistically creation and maintenance of such configurations in space dimensions $d = 1$ to 5 and find that they are easily created (maintained) for small (large) d. An intermediate d such as $d = 3$ seems to be ideal for the creation and maintenance of complex systems. This has consequences in a statistical description of the universe.

An effort has been made to classify complex configurations which often arise from simple algorithms [1], as well as to define measures of complexity valid for certain cases, for example, for a continuous stochastic signal $x(t)$ [2], or for a hierarchical structure such as a tree [3]. As in [1–3] we use a variable, which we call diversity in the following, for measuring the complexity of a system. Both a completely ordered configuration, such as that of a crystalline solid, and a completely disordered configuration, such as that of an ideal gas, are not really complex. However, the system passes through a very complex configuration in its transition from the former to the latter. The diversity first increases, then attains its maximum, when the system assumes the most complex configuration, and later it decreases again. The greater the diversity, the more complex the system is.

The term complexity can be associated with a variety of properties of a system. In many situations, complexity is associated with the diversity of size scales. In fluid mechanics, and in many other physical phenomena, complex behaviour is associated with a spatial inhomogeneity, i.e. with a diversity in size scales. An individual fractal or a fractal distribution of clusters are complex systems with diversified size scales [4]. In the present letter we shall consider only this concept of complexity, which covers a variety of situations of interest.

The observed diversity of a system cannot easily be accounted for by the dynamical equation it satisfies. Many aspects of the production and maintenance of large diversity can be explained by a statistical approach, rather than by a microscopic approach. The present letter attempts such a description of the formation and existence of a complex configuration starting from a simple configuration. Specifically, we deal with the numerical simulation of stochastic fragmentation and growth dynamics on large lattices. If the lifetime of individual fragments of the system is too short, such a configuration will either quickly form a large single object (in the case of aggregation) or decay rapidly to a gas or cloud

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state in which the only components are the building blocks (in the case of fragmentation). Both these final configurations are necessarily non-complex in nature. If, on the other hand, the stability of individual fragments is absolute, the complex system will rest indefinitely in an uninteresting configuration of minimal diversity. The interesting case is the one where there is a dynamical evolution, e.g. a configuration of large diversity is created from one of low diversity and survives for a long time and is eventually destroyed. We find that the probability of formation of a very complex configuration of clusters is exponentially reduced for a larger space dimension d for a fixed mass and the probability of survival of this configuration increases linearly with d. This indicates that $d = 3$ is perhaps optimal for the creation and maintenance of a complex system. As the universe is very complex, long-lived, and three-dimensional, this fact brings additional support to a statistical description of the clustering processes in the universe. The fractal distribution of the clusters of the universe and its fractal dimension has previously been explained by statistical arguments [5–7].

The concept of diversity is fundamental in an increasing number of contexts in the scientific literature, in connection with biological [8] and evolutionary [9] problems, self-organization, randomness [10], cellular automata [11], fractals [4, 12], and non-equilibrium phenomena [13]. Recently, the concept of diversity of sizes [13–16] has been extensively studied for several dissipative processes and cellular automata which generate a distribution of clusters. Such studies [13–16] cover situations of interest in physics, chemistry, and biology.

Diversity $D(t)$ which gives the number of different sizes at time t and is defined as

$$D(t) = \left\langle \sum_s \Theta[n(s,t)] \right\rangle \tag{1}$$

where $n(s,t)$ is the number of fragments of size (mass) s at time t, $\Theta(x) = 1$ if $x > 0$, and $= 0$ otherwise and the averaging $\langle\rangle$ is over different experiments. As in [2], the diversity of (1) can be considered as an integral over a coarse-grained distribution. The present diversity is also similar to the diversity introduced in [3].

We consider two fragmentation dynamics with (i) consumption of mass and with (ii) constant mass in addition to an aggregation dynamics with (iii) increase of mass. These dynamics, described below, are sufficiently simple to allow extensive numerical simulations and statistical analyses, yet sufficiently complex to exhibit a wide variety of complicated patterns.

(i) The particle at site i of an object S, of initial mass M_0 and average initial coordination q_0, on a d-dimensional lattice is selected at random. If the coordination q_i of i (i.e. the number of nearest neighbours of i) satisfies $q_i < q_{max} (\equiv 2d)$, then the particle at site i is consumed with probability p. Consumption of the particle at site i implies the decrease of the mass of S by one unit. Nothing happens if $q_i = q_{max}$. However, each random selection of a site i increases the time by one unit. This dynamics can simulate a number of phenomena such as the attack of a matrix by a kind of corrosive rain, or a pathogenic element (plague) acting on a tissue (plantation), and other phenomena of interest in physics, chemistry, biology, and ecology [13, 14].

(ii) The particle at site i of the object S of (i) is selected at random. If $q_i < q_{max}$ all bonds connecting the particle i to its nearest neighbours are broken with probability p. As a result a fragment of unit mass is generated and the initial mass is conserved. Nothing happens if $q_i = q_{max}$. Each random selection of a site i increases the time by one unit. The model (ii) can simulate fragmentation processes due, for example, to the mechanical, electrical, and chemical removal of bonds between sites of the object.

(iii) A site i on a d-dimensional lattice is chosen and occupied at random. Then another unoccupied site is chosen and occupied at random. Each random selection of a site i increases both the time and the mass by one unit. In this case M_0 and q_0 refer to the final mass and average coordination, when all sites are occupied. This model can simulate many aggregation processes such as deposition of particles in condensed matter physics, accretion in biology, ecology, or astrophysics.

With dynamics (i) and (ii) we performed numerical simulation employing massive Euclidean objects with initial mass M_0 as large as 10^7 for d varying from 1 to 5 and p varying from 0 to 1, as well as fractal objects with deterministic structures (Sierpinski carpets) and random porous structures (two-dimensional percolation clusters) with M_0 up to 3.2×10^6. With dynamics (iii) we considered aggregation on lattice of size up to 10^7. In each case an average over ten similar experiments were performed. These numerical simulations are unique at the present time considering the generality of the procedure and the very large objects employed.

In all three dynamics the system necessarily passes through the most complex configuration when diversity $D(t)$ attains its maximum D_{max}. The distribution of the fragments at the maximum of diversity for both the aggregation and fragmentation dynamics exhibit the fractal property or the complex nature with no preferred characteristic length [4]. The fractal nature of the cluster distribution is manifested through the well established scaling relation at the maximum of diversity [4, 12, 13, 17]

$$n(s) \sim s^{-\tau} \tag{2}$$

where $n(s)$ is the number of fragments of size s and τ is a critical exponent.

One of the most conspicuous findings of recent studies of cluster distribution [13–16] is the existence of the robust scaling relation $D_{max} \sim N_{max}^{1/2}$ between the maximum of diversity D_{max} and the maximum of the number of fragments N_{max} [13]. One also has the scaling relations $D_{max} \sim \rho(d)(M_0/q_0)^{1/2}$ and $N_{max} \sim g(d)(M_0/q_0)$ [13], where $\rho(d)$ and $g(d)$ are functions of the dimension d of space under consideration.

In order to quantify the production of diversity in different space dimensions from a certain fragmentation dynamics we consider the ratio

$$\rho(d) = D_{max}^2/(M_0/q_0)\,. \tag{3}$$

The function $\rho(d)$ provides a good measure of the intrinsic capacity of a system to generate diversity independent of its initial mass and coordination in a space of dimension d. The maintenance of diversity is quantified by the ratio

$$\epsilon \equiv \frac{\int_0^T D(t)\,\mathrm{d}t}{T D_{max}} = \frac{\bar{D}}{D_{max}} < 1 \tag{4}$$

where T is the time when diversity becomes unity. Here $\bar{D}$ is the average diversity and ϵ is essentially a normalized time-averaged value for $D(t)$. If ϵ is large, $D(t)$ maintains a value close to D_{max} for a long period of time; this means that the complex configuration survives for a long interval of time within the total 'life' T. For the same initial objects and for the same value of p, dynamics (i) and (ii) generate the same value for the ratios ρ and ϵ.

Figure 1 shows the exponential decay of the ratio $\rho(d)$ as d increases from 1 to 5 for models (i) and (ii). This plot refers to numerical simulation with both fractal and compact objects for $p = 0.2$ (□), 0.5 (○), and 0.8 (◇). In the plots the data for the largest objects are exhibited in each case. They are $L = 70\,000$ for $d = 1$, $L = 1000$ for $d = 2$, $L = 100$ for $d = 3$, $L = 40$ for $d = 4$, and $L = 25$ for $d = 5$. In addition, for $d = 2$, the symbols ▽, △ and ▷, respectively, refer to simulations on Sierpinski carpets of length $L = 2187$

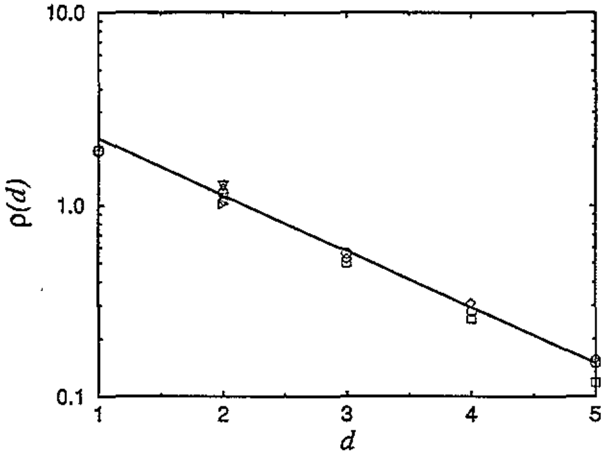

Figure 1. The ratio $\rho(d)$ versus d for fragmentation models (i) and (ii) with $p = 0.2$ (□), 0.5 (○), and 0.8 (◇) in space dimensions $d = 1$ to 5. In addition for d =2 there are three more points: Sierpinski carpet of size $L = 2187$ ($M_0 = 2\,097\,192$) for $p = 1$ (▽) and $p = 0$(△), and percolation cluster on lattice of $L = 2001$, for $p = 1$(▷). The full line is the fit given by (5). For details of the objects employed see text.

($M_0 = 2097\,152$) for $p = 1$ (▽) and $p = 0$ (△), and for percolation cluster on a lattice of length $L = 2001$, for $p = 1$ (▷). The data in figure 1 can be fitted to

$$\rho(d) = 4.3 \exp[-(0.67 \pm 0.05)d]\,. \tag{5}$$

In figure 2 we plot D^2_{max}/M_0 as a function of d for model (iii). In this aggregation model, unlike as in the fragmentation models considered in figure 1, since we are not involved with different geometries for the same value of d, the factor q_0 is omitted. In this case the lengths of lattices are $L = 70\,000$ ($d = 1$), $L = 1000$ ($d = 2$), $L = 100$ ($d = 3$), $L = 40$ ($d = 4$) and $L = 25(d = 5)$. The data in this figure can be fitted to

$$D^2_{max}/M_0 = 2.5 \exp[-(1.07 \pm 0.05)d]\,. \tag{6}$$

From figures 1 and 2 we find that a large diversity is easily formed for small d and q_0 and large M_0. This result is reasonable, although not obvious, since to generate many disconnected fragments or clusters from a single piece of matter one needs to cut progressively the (chemical) bonds that maintain matter concentrated over a small number of large clusters. These operations are more successful for small d because the number of bonds per site to be cut diminishes as d and q_0 decreases. Also, it seems quite reasonable to form large diversity for large objects.

In figure 3 we plot the ratio ϵ/d versus M_0 for models (i) and (ii). We find the following best fit in this case:

$$\epsilon/d = 0.61 M_0^{-0.23}\,. \tag{7}$$

For model (iii) (not shown in figure 3) we find

$$\epsilon/d = 480 M_0^{-0.64}\,. \tag{8}$$

In figure 3 we have three decades of mass scale with M_0 varying from 10^4 to 10^7. The larger the ratio ϵ, the higher is the stability of the complex configuration. Recalling that $D_{max} \sim (M_0/q_0)^{1/2}$ [13], it follows from (5) and (6) that, for a fixed q_0 and d, the larger the mass M_0, the more complex the system is. Consequently, from (7) and (8), we find

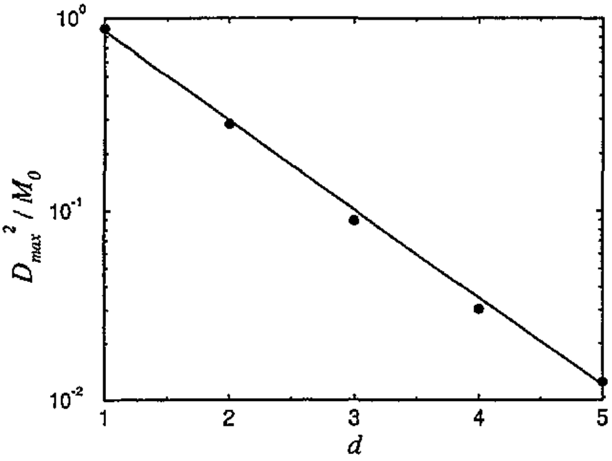

Figure 2. The ratio D^2_{max}/M_0 versus d for aggregation model (iii). The full line is the fit given by (6). For details of the objects employed see the text.

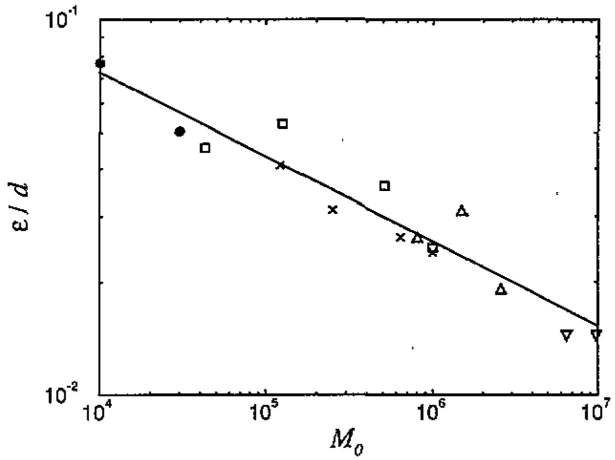

Figure 3. The ratio ϵ/d versus M_0 for models (i) and (ii) ($\bullet$: $d = 1$; $\times$: $d = 2$; $\square$: $d = 3$; $\triangle$: $d = 4$; $\triangledown$: $d = 5$). The full line refers to the fit given by (7).

that the more complex the system is, the more intense are the fluctuations that threaten its stability. Also, for a fixed M_0, stability increases linearly with d. This seems plausible, as *the stability of a complex configuration against the fluctuation or the noise is provided* by the average number of bonds per site, which increases with d. The same mechanism which creates efficiently a large diversity in smaller dimension, is also responsible for the quick destruction of the large diversity. There is a competition between stabilization through communication and instability through noise. The outcome of this competition determines the threshold of stability. In space of lower dimension a large diversity is easily generated but it is also quickly destroyed. This is quite reasonable, as to reduce a given diversity one needs to cut progressively the bonds between constituents of clusters, which is more efficiently performed in space of lower dimensionality.

The present study has some consequence in a possible statistical description of the universe. Usually, the evolution of the universe is described by quantum mechanics and cosmology [18]. However, certain observed properties about clustering in the universe, such as the observed fractal dimension ($\sim$1.2) [5–7] can be obtained in a stochastic statistical description. The same is true about the observed distribution of clusters of galaxies in the universe given by (2) with $\tau = 1.9$ [5–7]. The exponent $\tau = 1.9$ seems to be robust and observed in previous studies of fragmentation [13, 15] in three dimensions independent of the dynamics. Also, a remarkable characteristic of the universe is its capacity to generate a diversity of sizes from the microscale of elementary particles to the megascale of galaxies and clusters of galaxies in cosmology. As diversity in the three-dimensional universe is large and long-lived, the present study provides further evidence in favour of a statistical description of the clustering phenomena in the universe. This indicates that a statistical description may have a major role in interpreting the production and maintenance of diversity in the universe starting from a hot gaseous state right after the big bang.

In conclusion, from extensive numerical study of fragmentation and aggregation dynamics on large lattices we find that for fixed M_0 and q_0 the creation of diversity is attenuated exponentially for large d. The stability of the diversity increases linearly with d indicating that $d = 3$ is possibly optimal for the creation and maintenance of complex systems.

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